Mathematics That Works
Beyond the Blackboard

How to reach students when mathematics is not their only language

Tomasz Szemberg
Department of Mathematics, UKEN Krakow

University of the National Education Commission in Krakow

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The starting question

How can we make students experience mathematics as something that works?

$\forall \; \exists \; \Rightarrow$

For students of mathematics

Mathematics works as a language of

- proof,
- structure,
- abstraction.

$\int \quad f(x) \quad \Delta$

For students of engineering

Mathematics works as a language of

- modelling,
- computation,
- design.

Why this belongs at DIAM

DIAM asks how mathematics can be taught better
in the places where it is needed.

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Mathematics
as a discipline

$\longrightarrow$

Applications
in technical and
scientific subjects

$\star$

Innovation
AI, software, PBL,
active learning

My angle

Not: *blackboard* $\rightsquigarrow$ *screen*.

But: *content* $\rightsquigarrow$ *competence*.

A familiar diagnosis

What we often observe

- students wait for procedures,
- examples are solved, but not transferred,
- definitions are memorised, but not used,
- technology is either forbidden or overtrusted,
- assessment rewards reproduction.

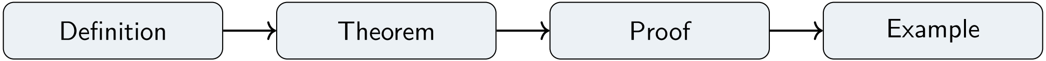
What students often need

- a reason to care,
- a first rough model,
- a chance to test ideas,
- language for explaining choices,
- criteria for correctness.

Take-away

The problem is often a missing bridge between examples and mathematical meaning.

The old implicit contract



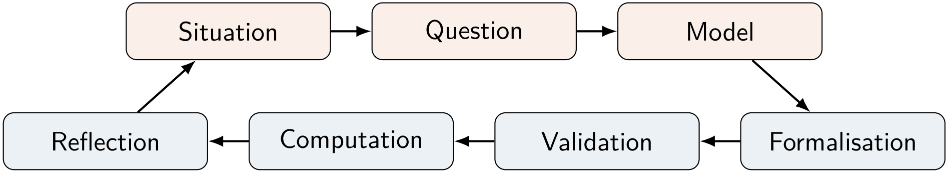
This path is not wrong

It is essential for mathematics students.
But for many first-year students it answers questions they have not yet learned to ask.

The hidden assumption

Students already know why the definition is worth having.

A different first move



Take-away

We do not abandon definitions, theorems and proofs. We create a need for them.

Four distances we must reduce

1. Distance of language

Students hear symbols before they hear meaning.

2. Distance of relevance

Students solve tasks without seeing the problem that produced them.

3. Distance of action

Students watch mathematics being done instead of doing mathematics.

4. Distance of tools

Students use powerful tools outside class, while class pretends those tools do not exist.

The aim is not entertainment. The aim is access to mathematical work.

Example: derivative before the definition

Start with a situation

A sensor records the temperature of a device.

Do not begin with the formal definition. Begin with a practical question:

When is the device changing too fast?

Then introduce mathematics

- average rate of change,
- units: degrees per second,
- slope of a secant line,
- shorter and shorter intervals,
- derivative as instantaneous rate.

Ask students to do

- look at a table or graph of $T(t)$,
- compare changes over equal time intervals,
- mark moments of rapid change,
- explain what “too fast” should mean.

Teacher’s move

The definition appears as a tool for answering a question students already understand.

Example: linear algebra beyond row reduction

Start with a situation

Several sensors measure the same process, but the readings contain noise.

How do we combine inconsistent information?

Ask students to do

- compare several noisy measurements,
- notice that the equations cannot all be exact,
- decide what a "best fit" should mean,
- discuss which information is independent.

Then introduce mathematics

- systems of equations,
- rank and dependence,
- the model $Ax \approx b$,
- least squares,
- projections and error.

Teacher's move

Row reduction becomes a method for deciding what information is independent and what conclusion is justified.

For mathematics students: proof also has to work

A proof is not only a text accepted by the teacher.

Why?

It explains why a statement is true.

Where?

It shows where the assumptions are used.

What next?

It makes the result transferable to a new situation.

Didactic consequence

Students should sometimes compare proofs, repair proofs, find missing assumptions, and translate a proof into a diagram or algorithm.

Technology: neither enemy nor saviour

Forbidden

The tool is treated as cheating. Students still use it, but without mathematical control.

Decorative

The tool appears in class, but the mathematical task is unchanged.

Integrated

The tool creates room for prediction, testing, visualisation, error analysis and explanation.

Take-away

A good digital task changes what students can investigate, not only how quickly they can calculate.

AI in a mathematics class: three useful roles

1. Critic

Ask AI for a solution and let students find mathematical and explanatory weaknesses.

2. Translator

Move between natural language, formula, graph, algorithm and interpretation.

3. Generator

Produce variants, examples and counterexamples which students must verify.

Non-negotiable principle

AI may support exploration, but responsibility for mathematical validity remains with the student.

Assessment: from knowing to using

Traditional signal	Competence signal	Possible evidence
Can reproduce a method	Chooses a method and justifies the choice	short explanation, comparison of approaches
Can compute correctly	Interprets the result and checks plausibility	units, limiting cases, numerical check
Knows definitions	Uses definitions to decide borderline cases	examples, non-examples, counterexamples
Can follow a proof	Can diagnose a proof	missing assumption, false step, repair

Take-away

Competence-oriented assessment does not lower standards. It makes standards more visible.

A 10-minute redesign template

1. Choose one concept students usually learn mechanically.
2. Write one real or realistic question that needs this concept.
3. Let students make a rough prediction before formal calculation.
4. Introduce the formal object as a tool for improving the prediction.
5. Use technology for exploration or verification, not as an answer machine.
6. End with a short explanation: What did the mathematics allow us to see?

Important

This can be done inside an ordinary lecture, exercise class or laboratory. It does not require every course to become a large project.

What changes for the teacher?

Less of

- explaining every step first,
- treating context as an afterthought,
- asking only for final answers,
- hiding uncertainty and failed attempts.

More of

- designing questions,
- making assumptions explicit,
- asking students to justify choices,
- discussing errors as mathematical data.

Take-away

The teacher remains the mathematical authority, but becomes also a designer of mathematical situations.

Three modest principles

1. Start not only from content, but from a mathematical need.
2. Let students act before they are fully fluent in the formal language.
3. Use technology to increase mathematical responsibility, not to remove it.

Mathematics works when students can use it to think.

Closing thought

Going beyond the blackboard does not mean leaving the blackboard behind.

It means that the blackboard is no longer the only place where mathematics happens.

Mathematics also happens when students model, test, compute, explain, doubt, correct and prove.

Thank you.

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