
Creating and solving list of exercises with Mathematica. Local extrema of a function in two variables.

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Introduction

We present a program in Mathematica that allows to create a long list of different exercises of similar types for students and the second one of answers.

The presentation is based on two examples. We want to investigate the local extrema of a function $f[a, b, c][x, y]$ in real variables x, y , depending on real parameters a, b, c . In both examples students have to solve by factorizing the system of two equation of the second degree. The choice of parameters should makes the numerical computations as easy as possible. Therefore, in the first example the parameters are chosen in such a way that the components of all stationary points are integers, while in the second one are integers or square roots of integers. Moreover, the resulting functions depend only on two parameters b, c .

For each example we create the set of about 30 exercises. This number can varies depending on needs. Thus the sets are suitable for on-line exams or home-works.

For better understanding, in Example 1 we explain, step by step, the result of each code we have created. In the Example 2 we present necessary codes in a compact form. We do not explain how Mathematica works! For that we refer to help menu in Mathematica.

The user can create her or his own list of parameters in the form `listofparameters = { {b1, c1}, {b2, c2}, {bm, cm} }` or try to replace the function f with her or his own one.

Each part runs independently from other.

For those not familiar with Mathematica the .pdf and .tex files are attached.

Finally we remark that problems no. 3261 and no. 3259, respectively 2986 and 2984 for the Polish edition, from the well- known problems book by G. N. Berman are the special cases of discussed functions.

Problem 1

Preliminaries

For students

Investigate the existence and type of local extrema of the function $f[b,c][x,y]$ in real variables x, y that depends on two real parameters b, c given by `listofparameters`. The list `listofparameters` can be of arbitrary length.

REMARK! The definition of function and `listofparameters` are the only quantities entered by the user.

Define the function.

In[]:=

```
f[b_, c_][x_, y_] := 3 b c x y + b x^2 y + c x y^2
```

For integers b, c the components of stationary points of the given function f are also integers. We define `listofparameters`, i.e. the set of pairs $\{b, c\}$.

In[]:=

```
listofparameters = { {-2, 1}, {-2, 2}, {1, 2}, {2, 1}, {2, 2}, {1, 3},
  {-1, 3}, {-3, 1}, {3, 1}, {2, 3}, {3, 2}, {-3, 2}, {-2, 3}, {-3, 3},
  {3, 3}, {1, 4}, {-4, 1}, {4, 1}, {4, 3}, {3, 4}, {-3, 4}, {3, -4},
  {-3, -4}, {-2, -4}, {2, 4}, {4, 2}, {4, 4}, {-4, 4}, {1, 5}, {5, 1}}
```

Out[]:=

```
{ {-2, 1}, {-2, 2}, {1, 2}, {2, 1}, {2, 2}, {1, 3}, {-1, 3},
  {-3, 1}, {3, 1}, {2, 3}, {3, 2}, {-3, 2}, {-2, 3}, {-3, 3},
  {3, 3}, {1, 4}, {-4, 1}, {4, 1}, {4, 3}, {3, 4}, {-3, 4}, {3, -4},
  {-3, -4}, {-2, -4}, {2, 4}, {4, 2}, {4, 4}, {-4, 4}, {1, 5}, {5, 1}}
```

We define the right hand sides of $f[b,c][x,y]$.

In[]:=

```
listofrighthandside = Apply[ f[#1, #2][x, y] &, listofparameters , {1}]
```

Out[]:=

```
{ -6 x y - 2 x^2 y + x y^2, -12 x y - 2 x^2 y + 2 x y^2, 6 x y + x^2 y + 2 x y^2, 6 x y + 2 x^2 y + x y^2,
  12 x y + 2 x^2 y + 2 x y^2, 9 x y + x^2 y + 3 x y^2, -9 x y - x^2 y + 3 x y^2, -9 x y - 3 x^2 y + x y^2,
  9 x y + 3 x^2 y + x y^2, 18 x y + 2 x^2 y + 3 x y^2, 18 x y + 3 x^2 y + 2 x y^2, -18 x y - 3 x^2 y + 2 x y^2,
  -18 x y - 2 x^2 y + 3 x y^2, -27 x y - 3 x^2 y + 3 x y^2, 27 x y + 3 x^2 y + 3 x y^2,
  12 x y + x^2 y + 4 x y^2, -12 x y - 4 x^2 y + x y^2, 12 x y + 4 x^2 y + x y^2, 36 x y + 4 x^2 y + 3 x y^2,
  36 x y + 3 x^2 y + 4 x y^2, -36 x y - 3 x^2 y + 4 x y^2, -36 x y + 3 x^2 y - 4 x y^2,
  36 x y - 3 x^2 y - 4 x y^2, 24 x y - 2 x^2 y - 4 x y^2, 24 x y + 2 x^2 y + 4 x y^2, 24 x y + 4 x^2 y + 2 x y^2,
  48 x y + 4 x^2 y + 4 x y^2, -48 x y - 4 x^2 y + 4 x y^2, 15 x y + x^2 y + 5 x y^2, 15 x y + 5 x^2 y + x y^2 }
```

The next two inputs make possible to write the left hand side of a function in the form $f[b,c][x,y]$.

In[]:=

```
listofheads = Apply[ f[#1, #2] &, listofparameters , {1}]
```

Out[]:=

```
{ f[-2, 1], f[-2, 2], f[1, 2], f[2, 1], f[2, 2], f[1, 3], f[-1, 3],
  f[-3, 1], f[3, 1], f[2, 3], f[3, 2], f[-3, 2], f[-2, 3], f[-3, 3],
  f[3, 3], f[1, 4], f[-4, 1], f[4, 1], f[4, 3], f[3, 4], f[-3, 4], f[3, -4],
  f[-3, -4], f[-2, -4], f[2, 4], f[4, 2], f[4, 4], f[-4, 4], f[1, 5], f[5, 1] }
```

In[]:=

```
listoflefthandside = Map[ HoldForm[# [x, y]] &, listofheads]
```

Out[]:=

```
{f[-2, 1] [x, y], f[-2, 2] [x, y], f[1, 2] [x, y], f[2, 1] [x, y], f[2, 2] [x, y],
 f[1, 3] [x, y], f[-1, 3] [x, y], f[-3, 1] [x, y], f[3, 1] [x, y], f[2, 3] [x, y],
 f[3, 2] [x, y], f[-3, 2] [x, y], f[-2, 3] [x, y], f[-3, 3] [x, y], f[3, 3] [x, y],
 f[1, 4] [x, y], f[-4, 1] [x, y], f[4, 1] [x, y], f[4, 3] [x, y], f[3, 4] [x, y],
 f[-3, 4] [x, y], f[3, -4] [x, y], f[-3, -4] [x, y], f[-2, -4] [x, y], f[2, 4] [x, y],
 f[4, 2] [x, y], f[4, 4] [x, y], f[-4, 4] [x, y], f[1, 5] [x, y], f[5, 1] [x, y]}
```

We obtain a numbered list of functions.

In[]:=

```
Do[ {Print[Style[r, Bold]],
      Print[ listoflefthandside[[r]] == ReleaseHold[ listoflefthandside[[r]] ] ] },
    {r, 1, Length[listoflefthandside]}]
```

1

$$f[-2, 1] [x, y] = -6xy - 2x^2y + xy^2$$

2

$$f[-2, 2] [x, y] = -12xy - 2x^2y + 2xy^2$$

3

$$f[1, 2] [x, y] = 6xy + x^2y + 2xy^2$$

4

$$f[2, 1] [x, y] = 6xy + 2x^2y + xy^2$$

5

$$f[2, 2] [x, y] = 12xy + 2x^2y + 2xy^2$$

6

$$f[1, 3] [x, y] = 9xy + x^2y + 3xy^2$$

7

$$f[-1, 3] [x, y] = -9xy - x^2y + 3xy^2$$

8

$$f[-3, 1] [x, y] = -9xy - 3x^2y + xy^2$$

9

$$f[3, 1] [x, y] = 9xy + 3x^2y + xy^2$$

10

$$f[2, 3] [x, y] = 18xy + 2x^2y + 3xy^2$$

11

$$f[3, 2] [x, y] = 18xy + 3x^2y + 2xy^2$$

12

$$f[-3, 2] [x, y] = -18xy - 3x^2y + 2xy^2$$

13

$$f[-2, 3] [x, y] = -18xy - 2x^2y + 3xy^2$$

14

$$f[-3, 3] [x, y] = -27xy - 3x^2y + 3xy^2$$

15

$$f[3, 3][x, y] = 27xy + 3x^2y + 3xy^2$$

16

$$f[1, 4][x, y] = 12xy + x^2y + 4xy^2$$

17

$$f[-4, 1][x, y] = -12xy - 4x^2y + xy^2$$

18

$$f[4, 1][x, y] = 12xy + 4x^2y + xy^2$$

19

$$f[4, 3][x, y] = 36xy + 4x^2y + 3xy^2$$

20

$$f[3, 4][x, y] = 36xy + 3x^2y + 4xy^2$$

21

$$f[-3, 4][x, y] = -36xy - 3x^2y + 4xy^2$$

22

$$f[3, -4][x, y] = -36xy + 3x^2y - 4xy^2$$

23

$$f[-3, -4][x, y] = 36xy - 3x^2y - 4xy^2$$

24

$$f[-2, -4][x, y] = 24xy - 2x^2y - 4xy^2$$

25

$$f[2, 4][x, y] = 24xy + 2x^2y + 4xy^2$$

26

$$f[4, 2][x, y] = 24xy + 4x^2y + 2xy^2$$

27

$$f[4, 4][x, y] = 48xy + 4x^2y + 4xy^2$$

28

$$f[-4, 4][x, y] = -48xy - 4x^2y + 4xy^2$$

29

$$f[1, 5][x, y] = 15xy + x^2y + 5xy^2$$

30

$$f[5, 1][x, y] = 15xy + 5x^2y + xy^2$$

Finally we get a list of exercises for students. To simplify the notation in what follows we write $f[x,y]$ rather than $f[b,c][x,y]$. We also attach an instruction for students (here in Polish).

In[]:=

```
Do[ { Print[Style[r, Bold]],
      Print["Zbadaj istnienie i rodzaj ekstremów lokalnych
            funkcji f zmiennych rzeczywistych x, y. Oblicz hesjan."],
      Print[ f[x, y] == ReleaseHold[ listoflefthandside[[r]]] ]},
    {r, 1, Length[listoflefthandside]}]
```

1

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -6xy - 2x^2y + xy^2$$

2

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -12xy - 2x^2y + 2xy^2$$

3

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 6xy + x^2y + 2xy^2$$

4

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 6xy + 2x^2y + xy^2$$

5

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 12xy + 2x^2y + 2xy^2$$

6

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 9xy + x^2y + 3xy^2$$

7

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -9xy - x^2y + 3xy^2$$

8

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -9xy - 3x^2y + xy^2$$

9

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 9xy + 3x^2y + xy^2$$

10

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 18xy + 2x^2y + 3xy^2$$

11

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 18xy + 3x^2y + 2xy^2$$

12

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -18xy - 3x^2y + 2xy^2$$

13

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -18xy - 2x^2y + 3xy^2$$

14

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -27xy - 3x^2y + 3xy^2$$

15

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 27xy + 3x^2y + 3xy^2$$

16

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 12xy + x^2y + 4xy^2$$

17

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -12xy - 4x^2y + xy^2$$

18

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 12xy + 4x^2y + xy^2$$

19

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 36xy + 4x^2y + 3xy^2$$

20

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 36xy + 3x^2y + 4xy^2$$

21

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -36xy - 3x^2y + 4xy^2$$

22

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -36xy + 3x^2y - 4xy^2$$

23

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 36xy - 3x^2y - 4xy^2$$

24

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 24xy - 2x^2y - 4xy^2$$

25

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 24xy + 2x^2y + 4xy^2$$

26

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 24xy + 4x^2y + 2xy^2$$

27

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 48xy + 4x^2y + 4xy^2$$

28

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = -48xy - 4x^2y + 4xy^2$$

29

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 15xy + x^2y + 5xy^2$$

30

Zbadaj istnienie i rodzaj ekstremów

lokalnych funkcji f zmiennych rzeczywistych x, y . Oblicz hesjan.

$$f[x, y] = 15xy + 5x^2y + xy^2$$

For teachers

From now on, we create a list of answers suitable for teachers.

lisofstationarypoints is a list of stationary points for successive functions.

In[]:=

```
lisofstationarypoints =
Table[ parametricsolution /. {b -> listofparameters[[r]][[1]],
  c -> listofparameters[[r]][[2]]}, {r, 1, Length[listofparameters]}]
```

Out[]:=

```
{{{x -> 0, y -> 6}, {x -> -1, y -> 2}, {x -> 0, y -> 0}, {x -> -3, y -> 0}},
 {{x -> 0, y -> 6}, {x -> -2, y -> 2}, {x -> 0, y -> 0}, {x -> -6, y -> 0}},
 {{x -> 0, y -> -3}, {x -> -2, y -> -1}, {x -> 0, y -> 0}, {x -> -6, y -> 0}},
 {{x -> 0, y -> -6}, {x -> -1, y -> -2}, {x -> 0, y -> 0}, {x -> -3, y -> 0}},
 {{x -> 0, y -> -6}, {x -> -2, y -> -2}, {x -> 0, y -> 0}, {x -> -6, y -> 0}},
 {{x -> 0, y -> -3}, {x -> -3, y -> -1}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> 3}, {x -> -3, y -> 1}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> 9}, {x -> -1, y -> 3}, {x -> 0, y -> 0}, {x -> -3, y -> 0}},
 {{x -> 0, y -> -9}, {x -> -1, y -> -3}, {x -> 0, y -> 0}, {x -> -3, y -> 0}},
 {{x -> 0, y -> -6}, {x -> -3, y -> -2}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> -9}, {x -> -2, y -> -3}, {x -> 0, y -> 0}, {x -> -6, y -> 0}},
 {{x -> 0, y -> 9}, {x -> -2, y -> 3}, {x -> 0, y -> 0}, {x -> -6, y -> 0}},
 {{x -> 0, y -> 6}, {x -> -3, y -> 2}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> 9}, {x -> -3, y -> 3}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> -9}, {x -> -3, y -> -3}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> -3}, {x -> -4, y -> -1}, {x -> 0, y -> 0}, {x -> -12, y -> 0}},
 {{x -> 0, y -> 12}, {x -> -1, y -> 4}, {x -> 0, y -> 0}, {x -> -3, y -> 0}},
 {{x -> 0, y -> -12}, {x -> -1, y -> -4}, {x -> 0, y -> 0}, {x -> -3, y -> 0}},
 {{x -> 0, y -> -12}, {x -> -3, y -> -4}, {x -> 0, y -> 0}, {x -> -9, y -> 0}},
 {{x -> 0, y -> -9}, {x -> -4, y -> -3}, {x -> 0, y -> 0}, {x -> -12, y -> 0}},
 {{x -> 0, y -> 9}, {x -> -4, y -> 3}, {x -> 0, y -> 0}, {x -> -12, y -> 0}},
 {{x -> 0, y -> -9}, {x -> 4, y -> -3}, {x -> 0, y -> 0}, {x -> 12, y -> 0}},
 {{x -> 0, y -> 9}, {x -> 4, y -> 3}, {x -> 0, y -> 0}, {x -> 12, y -> 0}},
 {{x -> 0, y -> 6}, {x -> 4, y -> 2}, {x -> 0, y -> 0}, {x -> 12, y -> 0}},
 {{x -> 0, y -> -6}, {x -> -4, y -> -2}, {x -> 0, y -> 0}, {x -> -12, y -> 0}},
 {{x -> 0, y -> -12}, {x -> -2, y -> -4}, {x -> 0, y -> 0}, {x -> -6, y -> 0}},
 {{x -> 0, y -> -12}, {x -> -4, y -> -4}, {x -> 0, y -> 0}, {x -> -12, y -> 0}},
 {{x -> 0, y -> 12}, {x -> -4, y -> 4}, {x -> 0, y -> 0}, {x -> -12, y -> 0}},
 {{x -> 0, y -> -3}, {x -> -5, y -> -1}, {x -> 0, y -> 0}, {x -> -15, y -> 0}},
 {{x -> 0, y -> -15}, {x -> -1, y -> -5}, {x -> 0, y -> 0}, {x -> -3, y -> 0}}]
```

list is the same list in a simplified form, suitable for further computations.

In[]:=

```
list = Map[ MapAt[ Last, #, {{1}, {2}}] &, lisofstationarypoints, {2}]
```

Out[]:=

```
{{{0, 6}, {-1, 2}, {0, 0}, {-3, 0}}, {{0, 6}, {-2, 2}, {0, 0}, {-6, 0}},
 {{0, -3}, {-2, -1}, {0, 0}, {-6, 0}}, {{0, -6}, {-1, -2}, {0, 0}, {-3, 0}},
 {{0, -6}, {-2, -2}, {0, 0}, {-6, 0}}, {{0, -3}, {-3, -1}, {0, 0}, {-9, 0}},
 {{0, 3}, {-3, 1}, {0, 0}, {-9, 0}}, {{0, 9}, {-1, 3}, {0, 0}, {-3, 0}},
 {{0, -9}, {-1, -3}, {0, 0}, {-3, 0}}, {{0, -6}, {-3, -2}, {0, 0}, {-9, 0}},
 {{0, -9}, {-2, -3}, {0, 0}, {-6, 0}}, {{0, 9}, {-2, 3}, {0, 0}, {-6, 0}},
 {{0, 6}, {-3, 2}, {0, 0}, {-9, 0}}, {{0, 9}, {-3, 3}, {0, 0}, {-9, 0}},
 {{0, -9}, {-3, -3}, {0, 0}, {-9, 0}}, {{0, -3}, {-4, -1}, {0, 0}, {-12, 0}},
 {{0, 12}, {-1, 4}, {0, 0}, {-3, 0}}, {{0, -12}, {-1, -4}, {0, 0}, {-3, 0}},
 {{0, -12}, {-3, -4}, {0, 0}, {-9, 0}}, {{0, -9}, {-4, -3}, {0, 0}, {-12, 0}},
 {{0, 9}, {-4, 3}, {0, 0}, {-12, 0}}, {{0, -9}, {4, -3}, {0, 0}, {12, 0}},
 {{0, 9}, {4, 3}, {0, 0}, {12, 0}}, {{0, 6}, {4, 2}, {0, 0}, {12, 0}},
 {{0, -6}, {-4, -2}, {0, 0}, {-12, 0}}, {{0, -12}, {-2, -4}, {0, 0}, {-6, 0}},
 {{0, -12}, {-4, -4}, {0, 0}, {-12, 0}}, {{0, 12}, {-4, 4}, {0, 0}, {-12, 0}},
 {{0, -3}, {-5, -1}, {0, 0}, {-15, 0}}, {{0, -15}, {-1, -5}, {0, 0}, {-3, 0}}]
```


We define the determinant of the Hesse matrix of the r -th function from `listofrighthandside` and $\frac{\partial^2 f}{\partial x^2}$ as well.

In[]:=

```
hesjan[r_][x_, y_] :=  
D[listofrighthandside[[r]], {x, 2}] × D[listofrighthandside[[r]], {y, 2}] -  
(D[listofrighthandside[[r]], x, y])^2
```

In[]:=

```
fbisx[r_][x_, y_] := D[listofrighthandside[[r]], {x, 2}]
```

`hesjanandfbisx` returns $\det H_2(f)$ and $\frac{\partial^2 f}{\partial x^2}$ for each function from `listofrighthandside`.

In[]:=

```
hesjanandfbisx =  
Table[{hesjan[r][x, y], fbisx[r][x, y]}, {r, 1, Length[listofrighthandside]}]
```

Out[]:=

```
{ {-8 x y - (-6 - 4 x + 2 y)^2, -4 y}, {-16 x y - (-12 - 4 x + 4 y)^2, -4 y},  
 {8 x y - (6 + 2 x + 4 y)^2, 2 y}, {8 x y - (6 + 4 x + 2 y)^2, 4 y},  
 {16 x y - (12 + 4 x + 4 y)^2, 4 y}, {12 x y - (9 + 2 x + 6 y)^2, 2 y},  
 {-12 x y - (-9 - 2 x + 6 y)^2, -2 y}, {-12 x y - (-9 - 6 x + 2 y)^2, -6 y},  
 {12 x y - (9 + 6 x + 2 y)^2, 6 y}, {24 x y - (18 + 4 x + 6 y)^2, 4 y},  
 {24 x y - (18 + 6 x + 4 y)^2, 6 y}, {-24 x y - (-18 - 6 x + 4 y)^2, -6 y},  
 {-24 x y - (-18 - 4 x + 6 y)^2, -4 y}, {-36 x y - (-27 - 6 x + 6 y)^2, -6 y},  
 {36 x y - (27 + 6 x + 6 y)^2, 6 y}, {16 x y - (12 + 2 x + 8 y)^2, 2 y},  
 {-16 x y - (-12 - 8 x + 2 y)^2, -8 y}, {16 x y - (12 + 8 x + 2 y)^2, 8 y},  
 {48 x y - (36 + 8 x + 6 y)^2, 8 y}, {48 x y - (36 + 6 x + 8 y)^2, 6 y},  
 {-48 x y - (-36 - 6 x + 8 y)^2, -6 y}, {-(-36 + 6 x - 8 y)^2 - 48 x y, 6 y},  
 {- (36 - 6 x - 8 y)^2 + 48 x y, -6 y}, {- (24 - 4 x - 8 y)^2 + 32 x y, -4 y},  
 {32 x y - (24 + 4 x + 8 y)^2, 4 y}, {32 x y - (24 + 8 x + 4 y)^2, 8 y},  
 {64 x y - (48 + 8 x + 8 y)^2, 8 y}, {-64 x y - (-48 - 8 x + 8 y)^2, -8 y},  
 {20 x y - (15 + 2 x + 10 y)^2, 2 y}, {20 x y - (15 + 10 x + 2 y)^2, 10 y}}
```

Next, we want to calculate the values of $\det H_2(f)$ and $\frac{\partial^2 f}{\partial x^2}$ for each function at each stationary point. This is given by values. In the example under consideration each function has four stationary points.

In[*]:=

```
hesjanandfbisxhasz = (hesjanandfbisx /. {x -> #1, y -> #2})
```

Out[*]=

```
{ {-8 #1 #2 - (-6 - 4 #1 + 2 #2)^2, -4 #2}, {-16 #1 #2 - (-12 - 4 #1 + 4 #2)^2, -4 #2},
{ 8 #1 #2 - (6 + 2 #1 + 4 #2)^2, 2 #2}, { 8 #1 #2 - (6 + 4 #1 + 2 #2)^2, 4 #2},
{16 #1 #2 - (12 + 4 #1 + 4 #2)^2, 4 #2}, {12 #1 #2 - (9 + 2 #1 + 6 #2)^2, 2 #2},
{-12 #1 #2 - (-9 - 2 #1 + 6 #2)^2, -2 #2}, {-12 #1 #2 - (-9 - 6 #1 + 2 #2)^2, -6 #2},
{12 #1 #2 - (9 + 6 #1 + 2 #2)^2, 6 #2}, {24 #1 #2 - (18 + 4 #1 + 6 #2)^2, 4 #2},
{24 #1 #2 - (18 + 6 #1 + 4 #2)^2, 6 #2}, {-24 #1 #2 - (-18 - 6 #1 + 4 #2)^2, -6 #2},
{-24 #1 #2 - (-18 - 4 #1 + 6 #2)^2, -4 #2}, {-36 #1 #2 - (-27 - 6 #1 + 6 #2)^2, -6 #2},
{36 #1 #2 - (27 + 6 #1 + 6 #2)^2, 6 #2}, {16 #1 #2 - (12 + 2 #1 + 8 #2)^2, 2 #2},
{-16 #1 #2 - (-12 - 8 #1 + 2 #2)^2, -8 #2}, {16 #1 #2 - (12 + 8 #1 + 2 #2)^2, 8 #2},
{48 #1 #2 - (36 + 8 #1 + 6 #2)^2, 8 #2}, {48 #1 #2 - (36 + 6 #1 + 8 #2)^2, 6 #2},
{-48 #1 #2 - (-36 - 6 #1 + 8 #2)^2, -6 #2}, {-(-36 + 6 #1 - 8 #2)^2 - 48 #1 #2, 6 #2},
{-(36 - 6 #1 - 8 #2)^2 + 48 #1 #2, -6 #2}, {-(24 - 4 #1 - 8 #2)^2 + 32 #1 #2, -4 #2},
{32 #1 #2 - (24 + 4 #1 + 8 #2)^2, 4 #2}, {32 #1 #2 - (24 + 8 #1 + 4 #2)^2, 8 #2},
{64 #1 #2 - (48 + 8 #1 + 8 #2)^2, 8 #2}, {-64 #1 #2 - (-48 - 8 #1 + 8 #2)^2, -8 #2},
{20 #1 #2 - (15 + 2 #1 + 10 #2)^2, 2 #2}, {20 #1 #2 - (15 + 10 #1 + 2 #2)^2, 10 #2}}
```

In[]:=

```
values = Table[Apply[Evaluate[hessianandfbisxhasz[[r]]] &, list[[r]], {1}],
  {r, 1, Length[listofrighthandside]}]
```

Out[]:=

```
{{{-36, -24}, {12, -8}, {-36, 0}, {-36, 0}},
 {{-144, -24}, {48, -8}, {-144, 0}, {-144, 0}},
 {{-36, -6}, {12, -2}, {-36, 0}, {-36, 0}},
 {{-36, -24}, {12, -8}, {-36, 0}, {-36, 0}},
 {{-144, -24}, {48, -8}, {-144, 0}, {-144, 0}},
 {{-81, -6}, {27, -2}, {-81, 0}, {-81, 0}},
 {{-81, -6}, {27, -2}, {-81, 0}, {-81, 0}},
 {{-81, -54}, {27, -18}, {-81, 0}, {-81, 0}},
 {{-81, -54}, {27, -18}, {-81, 0}, {-81, 0}},
 {{-324, -24}, {108, -8}, {-324, 0}, {-324, 0}},
 {{-324, -54}, {108, -18}, {-324, 0}, {-324, 0}},
 {{-324, -54}, {108, -18}, {-324, 0}, {-324, 0}},
 {{-324, -24}, {108, -8}, {-324, 0}, {-324, 0}},
 {{-729, -54}, {243, -18}, {-729, 0}, {-729, 0}},
 {{-729, -54}, {243, -18}, {-729, 0}, {-729, 0}},
 {{-144, -6}, {48, -2}, {-144, 0}, {-144, 0}},
 {{-144, -96}, {48, -32}, {-144, 0}, {-144, 0}},
 {{-144, -96}, {48, -32}, {-144, 0}, {-144, 0}},
 {{-1296, -96}, {432, -32}, {-1296, 0}, {-1296, 0}},
 {{-1296, -54}, {432, -18}, {-1296, 0}, {-1296, 0}},
 {{-1296, -54}, {432, -18}, {-1296, 0}, {-1296, 0}},
 {{-1296, -54}, {432, -18}, {-1296, 0}, {-1296, 0}},
 {{-1296, -54}, {432, -18}, {-1296, 0}, {-1296, 0}},
 {{-576, -24}, {192, -8}, {-576, 0}, {-576, 0}},
 {{-576, -24}, {192, -8}, {-576, 0}, {-576, 0}},
 {{-576, -96}, {192, -32}, {-576, 0}, {-576, 0}},
 {{-2304, -96}, {768, -32}, {-2304, 0}, {-2304, 0}},
 {{-2304, -96}, {768, -32}, {-2304, 0}, {-2304, 0}},
 {{-225, -6}, {75, -2}, {-225, 0}, {-225, 0}},
 {{-225, -150}, {75, -50}, {-225, 0}, {-225, 0}}}
```

Finally, we create a list of answers.

In[]:=

```
Do[{Print[Style[r, 18, Red]], Print["Function:"],
  Print[ listoflefthandside[[r]] == ReleaseHold[ listoflefthandside[[r]] ] ],
  Print["Stationary points:"], Print[ list[[r]] ], Print["Hesse det and  $\frac{\partial^2 f}{\partial x^2}$ :"],
  Print[hessianandfbisx[[r]] ], Print["Values at stationary points:"],
  Print[ values[[r]] ]}], {r, 1, Length[listoflefthandside]}]
```

1

Function:

$$f[-2, 1][x, y] = -6xy - 2x^2y + xy^2$$

Stationary points:

$$\{\{0, 6\}, \{-1, 2\}, \{0, 0\}, \{-3, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-8xy - (-6 - 4x + 2y)^2, -4y\}$$

Values at stationary points:

$$\{-36, -24\}, \{12, -8\}, \{-36, 0\}, \{-36, 0\}$$

2

Function:

$$f[-2, 2][x, y] = -12xy - 2x^2y + 2xy^2$$

Stationary points:

$$\{0, 6\}, \{-2, 2\}, \{0, 0\}, \{-6, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-16xy - (-12 - 4x + 4y)^2, -4y\}$$

Values at stationary points:

$$\{-144, -24\}, \{48, -8\}, \{-144, 0\}, \{-144, 0\}$$

3

Function:

$$f[1, 2][x, y] = 6xy + x^2y + 2xy^2$$

Stationary points:

$$\{0, -3\}, \{-2, -1\}, \{0, 0\}, \{-6, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{8xy - (6 + 2x + 4y)^2, 2y\}$$

Values at stationary points:

$$\{-36, -6\}, \{12, -2\}, \{-36, 0\}, \{-36, 0\}$$

4

Function:

$$f[2, 1][x, y] = 6xy + 2x^2y + xy^2$$

Stationary points:

$$\{0, -6\}, \{-1, -2\}, \{0, 0\}, \{-3, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{8xy - (6 + 4x + 2y)^2, 4y\}$$

Values at stationary points:

$$\{-36, -24\}, \{12, -8\}, \{-36, 0\}, \{-36, 0\}$$

5

Function:

$$f[2, 2][x, y] = 12xy + 2x^2y + 2xy^2$$

Stationary points:

$$\{0, -6\}, \{-2, -2\}, \{0, 0\}, \{-6, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{16xy - (12 + 4x + 4y)^2, 4y\}$$

Values at stationary points:

$$\{-144, -24\}, \{48, -8\}, \{-144, 0\}, \{-144, 0\}$$

6

Function:

$$f[1, 3][x, y] = 9xy + x^2y + 3xy^2$$

Stationary points:

$$\{0, -3\}, \{-3, -1\}, \{0, 0\}, \{-9, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{12xy - (9 + 2x + 6y)^2, 2y\}$$

Values at stationary points:

$$\{-81, -6\}, \{27, -2\}, \{-81, 0\}, \{-81, 0\}$$

7

Function:

$$f[-1, 3][x, y] = -9xy - x^2y + 3xy^2$$

Stationary points:

$$\{0, 3\}, \{-3, 1\}, \{0, 0\}, \{-9, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-12xy - (-9 - 2x + 6y)^2, -2y\}$$

Values at stationary points:

$$\{-81, -6\}, \{27, -2\}, \{-81, 0\}, \{-81, 0\}$$

8

Function:

$$f[-3, 1][x, y] = -9xy - 3x^2y + xy^2$$

Stationary points:

$$\{0, 9\}, \{-1, 3\}, \{0, 0\}, \{-3, 0\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-12xy - (-9 - 6x + 2y)^2, -6y\}$$

Values at stationary points:

$$\{-81, -54\}, \{27, -18\}, \{-81, 0\}, \{-81, 0\}$$

9

Function:

$$f[3, 1][x, y] = 9xy + 3x^2y + xy^2$$

Stationary points:

$$\{\{0, -9\}, \{-1, -3\}, \{0, 0\}, \{-3, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{12xy - (9 + 6x + 2y)^2, 6y\}$$

Values at stationary points:

$$\{\{-81, -54\}, \{27, -18\}, \{-81, 0\}, \{-81, 0\}\}$$

10

Function:

$$f[2, 3][x, y] = 18xy + 2x^2y + 3xy^2$$

Stationary points:

$$\{\{0, -6\}, \{-3, -2\}, \{0, 0\}, \{-9, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{24xy - (18 + 4x + 6y)^2, 4y\}$$

Values at stationary points:

$$\{\{-324, -24\}, \{108, -8\}, \{-324, 0\}, \{-324, 0\}\}$$

11

Function:

$$f[3, 2][x, y] = 18xy + 3x^2y + 2xy^2$$

Stationary points:

$$\{\{0, -9\}, \{-2, -3\}, \{0, 0\}, \{-6, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{24xy - (18 + 6x + 4y)^2, 6y\}$$

Values at stationary points:

$$\{\{-324, -54\}, \{108, -18\}, \{-324, 0\}, \{-324, 0\}\}$$

12

Function:

$$f[-3, 2][x, y] = -18xy - 3x^2y + 2xy^2$$

Stationary points:

$$\{\{0, 9\}, \{-2, 3\}, \{0, 0\}, \{-6, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-24xy - (-18 - 6x + 4y)^2, -6y\}$$

Values at stationary points:

$$\{\{-324, -54\}, \{108, -18\}, \{-324, 0\}, \{-324, 0\}\}$$

13

Function:

$$f[-2, 3][x, y] = -18xy - 2x^2y + 3xy^2$$

Stationary points:

$$\{\{0, 6\}, \{-3, 2\}, \{0, 0\}, \{-9, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-24xy - (-18 - 4x + 6y)^2, -4y\}$$

Values at stationary points:

$$\{\{-324, -24\}, \{108, -8\}, \{-324, 0\}, \{-324, 0\}\}$$

14

Function:

$$f[-3, 3][x, y] = -27xy - 3x^2y + 3xy^2$$

Stationary points:

$$\{\{0, 9\}, \{-3, 3\}, \{0, 0\}, \{-9, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-36xy - (-27 - 6x + 6y)^2, -6y\}$$

Values at stationary points:

$$\{\{-729, -54\}, \{243, -18\}, \{-729, 0\}, \{-729, 0\}\}$$

15

Function:

$$f[3, 3][x, y] = 27xy + 3x^2y + 3xy^2$$

Stationary points:

$$\{\{0, -9\}, \{-3, -3\}, \{0, 0\}, \{-9, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{36xy - (27 + 6x + 6y)^2, 6y\}$$

Values at stationary points:

$$\{\{-729, -54\}, \{243, -18\}, \{-729, 0\}, \{-729, 0\}\}$$

16

Function:

$$f[1, 4][x, y] = 12xy + x^2y + 4xy^2$$

Stationary points:

$$\{\{0, -3\}, \{-4, -1\}, \{0, 0\}, \{-12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{16xy - (12 + 2x + 8y)^2, 2y\}$$

Values at stationary points:

$$\{\{-144, -6\}, \{48, -2\}, \{-144, 0\}, \{-144, 0\}\}$$

17

Function:

$$f[-4, 1][x, y] = -12xy - 4x^2y + xy^2$$

Stationary points:

$$\{\{0, 12\}, \{-1, 4\}, \{0, 0\}, \{-3, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-16xy - (-12 - 8x + 2y)^2, -8y\}$$

Values at stationary points:

$$\{\{-144, -96\}, \{48, -32\}, \{-144, 0\}, \{-144, 0\}\}$$

18

Function:

$$f[4, 1][x, y] = 12xy + 4x^2y + xy^2$$

Stationary points:

$$\{\{0, -12\}, \{-1, -4\}, \{0, 0\}, \{-3, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{16xy - (12 + 8x + 2y)^2, 8y\}$$

Values at stationary points:

$$\{\{-144, -96\}, \{48, -32\}, \{-144, 0\}, \{-144, 0\}\}$$

19

Function:

$$f[4, 3][x, y] = 36xy + 4x^2y + 3xy^2$$

Stationary points:

$$\{\{0, -12\}, \{-3, -4\}, \{0, 0\}, \{-9, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{48xy - (36 + 8x + 6y)^2, 8y\}$$

Values at stationary points:

$$\{\{-1296, -96\}, \{432, -32\}, \{-1296, 0\}, \{-1296, 0\}\}$$

20

Function:

$$f[3, 4][x, y] = 36xy + 3x^2y + 4xy^2$$

Stationary points:

$$\{\{0, -9\}, \{-4, -3\}, \{0, 0\}, \{-12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{48xy - (36 + 6x + 8y)^2, 6y\}$$

Values at stationary points:

$$\{\{-1296, -54\}, \{432, -18\}, \{-1296, 0\}, \{-1296, 0\}\}$$

21

Function:

$$f[-3, 4][x, y] = -36xy - 3x^2y + 4xy^2$$

Stationary points:

$$\{\{0, 9\}, \{-4, 3\}, \{0, 0\}, \{-12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-48xy - (-36 - 6x + 8y)^2, -6y\}$$

Values at stationary points:

$$\{\{-1296, -54\}, \{432, -18\}, \{-1296, 0\}, \{-1296, 0\}\}$$

22

Function:

$$f[3, -4][x, y] = -36xy + 3x^2y - 4xy^2$$

Stationary points:

$$\{\{0, -9\}, \{4, -3\}, \{0, 0\}, \{12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-(-36 + 6x - 8y)^2 - 48xy, 6y\}$$

Values at stationary points:

$$\{\{-1296, -54\}, \{432, -18\}, \{-1296, 0\}, \{-1296, 0\}\}$$

23

Function:

$$f[-3, -4][x, y] = 36xy - 3x^2y - 4xy^2$$

Stationary points:

$$\{\{0, 9\}, \{4, 3\}, \{0, 0\}, \{12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-(36 - 6x - 8y)^2 + 48xy, -6y\}$$

Values at stationary points:

$$\{\{-1296, -54\}, \{432, -18\}, \{-1296, 0\}, \{-1296, 0\}\}$$

24

Function:

$$f[-2, -4][x, y] = 24xy - 2x^2y - 4xy^2$$

Stationary points:

$$\{\{0, 6\}, \{4, 2\}, \{0, 0\}, \{12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-(24 - 4x - 8y)^2 + 32xy, -4y\}$$

Values at stationary points:

$$\{\{-576, -24\}, \{192, -8\}, \{-576, 0\}, \{-576, 0\}\}$$

25

Function:

$$f[2, 4][x, y] = 24xy + 2x^2y + 4xy^2$$

Stationary points:

$$\{\{0, -6\}, \{-4, -2\}, \{0, 0\}, \{-12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{32xy - (24 + 4x + 8y)^2, 4y\}$$

Values at stationary points:

$$\{\{-576, -24\}, \{192, -8\}, \{-576, 0\}, \{-576, 0\}\}$$

26

Function:

$$f[4, 2][x, y] = 24xy + 4x^2y + 2xy^2$$

Stationary points:

$$\{\{0, -12\}, \{-2, -4\}, \{0, 0\}, \{-6, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{32xy - (24 + 8x + 4y)^2, 8y\}$$

Values at stationary points:

$$\{\{-576, -96\}, \{192, -32\}, \{-576, 0\}, \{-576, 0\}\}$$

27

Function:

$$f[4, 4][x, y] = 48xy + 4x^2y + 4xy^2$$

Stationary points:

$$\{\{0, -12\}, \{-4, -4\}, \{0, 0\}, \{-12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{64xy - (48 + 8x + 8y)^2, 8y\}$$

Values at stationary points:

$$\{\{-2304, -96\}, \{768, -32\}, \{-2304, 0\}, \{-2304, 0\}\}$$

28

Function:

$$f[-4, 4][x, y] = -48xy - 4x^2y + 4xy^2$$

Stationary points:

$$\{\{0, 12\}, \{-4, 4\}, \{0, 0\}, \{-12, 0\}\}$$

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{-64xy - (-48 - 8x + 8y)^2, -8y\}$$

Values at stationary points:

```
{{-2304, -96}, {768, -32}, {-2304, 0}, {-2304, 0}}
```

29

Function:

$$f[1, 5][x, y] = 15xy + x^2y + 5xy^2$$

Stationary points:

```
{{0, -3}, {-5, -1}, {0, 0}, {-15, 0}}
```

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{20xy - (15 + 2x + 10y)^2, 2y\}$$

Values at stationary points:

```
{{-225, -6}, {75, -2}, {-225, 0}, {-225, 0}}
```

30

Function:

$$f[5, 1][x, y] = 15xy + 5x^2y + xy^2$$

Stationary points:

```
{{0, -15}, {-1, -5}, {0, 0}, {-3, 0}}
```

Hesse det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{20xy - (15 + 10x + 2y)^2, 10y\}$$

Values at stationary points:

```
{{-225, -150}, {75, -50}, {-225, 0}, {-225, 0}}
```

Clear all definitions

In[]:=

```
Clear[f, listofparameters, listofrighthandside,
listofheads, listoflefthandside, listofstationarypoints, list,
fbisx, hesjan, hesjanandfbisx, hesjanandfbisxhasz, values]
```

Problem 2

Preliminaries

Define the function $f[x, y] = ax^3 + bx^2y + cx^2 + dy^2$, where a, b, c, d , are real parameters. We want to investigate the local extrema of f .

In[]:=

```
f[x_, y_] := a x^3 + b x y^2 + c x^2 + d y^2
```

For a given values of a, b, c, d we want to make the numerical computations as simply as possible.

In[]:=

```
Solve[ D[f[x, y], x] == 0 && D[f[x, y], y] == 0 &&
a != 0 && b != 0 && c != 0 && d != 0, {x, y}]
```

Out[]:=

$$\left\{ \left\{ x \rightarrow 0, y \rightarrow 0 \right\}, \left\{ x \rightarrow -\frac{2c}{3a}, y \rightarrow 0 \right\}, \right. \\ \left. \left\{ x \rightarrow -\frac{d}{b}, y \rightarrow -\frac{\sqrt{2bcd - 3ad^2}}{b^{3/2}} \right\}, \left\{ x \rightarrow -\frac{d}{b}, y \rightarrow \frac{\sqrt{2bcd - 3ad^2}}{b^{3/2}} \right\} \right\}$$

Suppose $3a|c$ and $b|d$ and put

In[]:=

```
f[x_, y_] := a x^3 + b x y^2 + c 3 a x^2 + d b y^2
```

In[]:=

```
Solve[ D[f[x, y], x] == 0 && D[f[x, y], y] == 0 &&
a != 0 && b != 0 && c != 0 && d != 0, {x, y}]
```

Out[]:=

$$\left\{ \left\{ x \rightarrow 0, y \rightarrow 0 \right\}, \left\{ x \rightarrow -2c, y \rightarrow 0 \right\}, \right. \\ \left. \left\{ x \rightarrow -d, y \rightarrow -\frac{\sqrt{3} \sqrt{2acd - ad^2}}{\sqrt{b}} \right\}, \left\{ x \rightarrow -d, y \rightarrow \frac{\sqrt{3} \sqrt{2acd - ad^2}}{\sqrt{b}} \right\} \right\}$$

Finally suppose $b|ad$ and put

In[]:=

```
f[x_, y_] := a x^3 + a d x y^2 + c 3 a x^2 + d a d y^2
```

In[]:=

```
Solve[ D[f[x, y], x] == 0 && D[f[x, y], y] == 0 &&
a != 0 && b != 0 && c != 0 && d != 0, {x, y}]
```

Out[]:=

$$\left\{ \left\{ x \rightarrow -d, y \rightarrow -\sqrt{3} \sqrt{2c-d} \right\}, \right. \\ \left. \left\{ x \rightarrow -d, y \rightarrow \sqrt{3} \sqrt{2c-d} \right\}, \left\{ x \rightarrow 0, y \rightarrow 0 \right\}, \left\{ x \rightarrow -2c, y \rightarrow 0 \right\} \right\}$$

Now, if c, d are integers, then the coordinates of stationary points are integers and the square roots of them.

In[]:=

```
parametricssolution = Solve[ D[f[x, y], x] == 0 &&
D[f[x, y], y] == 0 && a != 0 && b != 0 && c != 0 && d != 0, {x, y}]
```

Out[]:=

$$\left\{ \left\{ x \rightarrow -d, y \rightarrow -\sqrt{3} \sqrt{2c-d} \right\}, \right. \\ \left. \left\{ x \rightarrow -d, y \rightarrow \sqrt{3} \sqrt{2c-d} \right\}, \left\{ x \rightarrow 0, y \rightarrow 0 \right\}, \left\{ x \rightarrow -2c, y \rightarrow 0 \right\} \right\}$$

In[]:=

```
Clear[f]
```

For students

In what follows, we deal with the function $f[c,d][x,y] = x^3 + d x y^2 + 3 c x^2 + d^2 y^2$, with non-zero c, d .

In[]:=

```
f[c_, d_] [x_, y_] := x^3 + d x y^2 + 3 c x^2 + d^2 y^2
```

Moreover, we have to define our own list of parameters listofparameters of arbitrary length. For example

```
In[ ]:= listofparameters = { {2, -1}, {1, 2}, {2, 1}, {1, -2}, {2, 1}, {2, 2},
    {2, -2}, {3, 1}, {3, -1}, {2, 3}, {3, 2}, {3, 3}, {4, 1}, {4, -1},
    {4, 3}, {3, 4}, {2, 4}, {4, -3}, {3, -4}, {-2, -4}, {4, 2}, {4, 2},
    {4, -2}, {4, 4}, {4, -4}, {5, -1}, {5, 1}, {5, 2}, {5, 3}, {5, 4}}

Out[ ]:= { {2, -1}, {1, 2}, {2, 1}, {1, -2}, {2, 1}, {2, 2}, {2, -2}, {3, 1}, {3, -1}, {2, 3},
    {3, 2}, {3, 3}, {4, 1}, {4, -1}, {4, 3}, {3, 4}, {2, 4}, {4, -3}, {3, -4}, {-2, -4},
    {4, 2}, {4, 2}, {4, -2}, {4, 4}, {4, -4}, {5, -1}, {5, 1}, {5, 2}, {5, 3}, {5, 4}}
```

The length of listofparameters is

```
In[ ]:= Length[listofparameters]
```

```
Out[ ]:= 30
```

From now on we write $f[x,y]$ rather than $f[c,d][x,y]$.

We define two function that act on listofparameters. The first one returns a list of exercises for students together with instruction. The second one returns a list of respective answers needed for a teacher.

```
In[ ]:= ForStudents[xxx_] := Module[{ listofrighthandside, listofheads,
    listoflefthandside,
    listofrighthandside = Apply[ f[#1, #2][x, y] &, xxx, {1}];
    listofheads = Apply[ f[#1, #2] &, xxx, {1}];
    listoflefthandside = Map[ HoldForm[# [x, y]] &, listofheads];
    Do[{Print[Style[r, Bold]], Print["Dana jest funkcja f zmiennych x, y, zależna
        od dwóch parametrów c, d:  $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$ .
        Zbadaj istnienie ekstremów lokalnych funkcji f. Oblicz hesjan."],
        Print[ f[x, y] == ReleaseHold[ listoflefthandside[[r]]]]},
    {r, 1, Length[listoflefthandside]}}]
```

```
In[ ]:= ForStudents[listofparameters]
```

1

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6 x^2 + x^3 + y^2 - x y^2$$

2

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 3 x^2 + x^3 + 4 y^2 + 2 x y^2$$

3

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6 x^2 + x^3 + y^2 + x y^2$$

4

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 3x^2 + x^3 + 4y^2 - 2xy^2$$

5

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6x^2 + x^3 + y^2 + xy^2$$

6

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6x^2 + x^3 + 4y^2 + 2xy^2$$

7

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6x^2 + x^3 + 4y^2 - 2xy^2$$

8

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 9x^2 + x^3 + y^2 + xy^2$$

9

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 9x^2 + x^3 + y^2 - xy^2$$

10

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6x^2 + x^3 + 9y^2 + 3xy^2$$

11

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 9x^2 + x^3 + 4y^2 + 2xy^2$$

12

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 9x^2 + x^3 + 9y^2 + 3xy^2$$

13

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12x^2 + x^3 + y^2 + xy^2$$

14

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x, y] == x^3 + dxy^2 + 3cx^2 + d^2y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12x^2 + x^3 + y^2 - xy^2$$

15

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 9 y^2 + 3 x y^2$$

16

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 9 x^2 + x^3 + 16 y^2 + 4 x y^2$$

17

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 6 x^2 + x^3 + 16 y^2 + 4 x y^2$$

18

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 9 y^2 - 3 x y^2$$

19

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 9 x^2 + x^3 + 16 y^2 - 4 x y^2$$

20

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == -6 x^2 + x^3 + 16 y^2 - 4 x y^2$$

21

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 4 y^2 + 2 x y^2$$

22

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 4 y^2 + 2 x y^2$$

23

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 4 y^2 - 2 x y^2$$

24

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 16 y^2 + 4 x y^2$$

25

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 12 x^2 + x^3 + 16 y^2 - 4 x y^2$$

26

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 15 x^2 + x^3 + y^2 - x y^2$$

27

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 15 x^2 + x^3 + y^2 + x y^2$$

28

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 15 x^2 + x^3 + 4 y^2 + 2 x y^2$$

29

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 15 x^2 + x^3 + 9 y^2 + 3 x y^2$$

30

Dana jest funkcja f zmiennych x, y , zależna od dwóch parametrów c, d : $f[x,y] == x^3 + d x y^2 + 3 c x^2 + d^2 y^2$. Zbadaj istnienie ekstremów lokalnych funkcji f . Oblicz hesjan.

$$f[x, y] == 15 x^2 + x^3 + 16 y^2 + 4 x y^2$$

For teachers

In[]:=

```

ForTeachers[xxx_] :=
Module[ { parametricsolution, lisofstationarypoints, list, listofrighthandside,
  listofheads, listoflefthandside, hesjanandfbisx, hesjanandfbisxhasz,
  values
}, parametricsolution = Solve[ D[f[c, d][x, y], x] == 0 &&
  D[f[c, d][x, y], y] == 0 && a != 0 && b != 0 && c != 0 && d != 0, {x, y}];
lisofstationarypoints = Table[parametricsolution /.
  {c -> xxx[[r]][[1]], d -> xxx[[r]][[2]]}, {r, 1, Length[xxx]}];
list = Map[MapAt[Last, #, {{1}, {2}}] &, lisofstationarypoints, {2}];
"listofparameters is crated by the user";
listofrighthandside = Apply[ f[#1, #2][x, y] &, xxx, {1}];
listofheads = Apply[ f[#1, #2] &, xxx, {1}];
listoflefthandside = Map[ HoldForm[# [x, y]] &, listofheads];
hesjan[r_][x_, y_] := D[listofrighthandside[[r]], {x, 2}] ×
  D[listofrighthandside[[r]], {y, 2}] - (D[listofrighthandside[[r]], x, y])^2;
fbisx[r_][x_, y_] := D[listofrighthandside[[r]], {x, 2}];
hesjanandfbisx = Table[{ hesjan[r][x, y], fbisx[r][x, y]},
  {r, 1, Length[listofrighthandside]}];
hesjanandfbisxhasz = (hesjanandfbisx /. {x -> #1, y -> #2});

"values returns values of Hessian det and  $\frac{\partial^2 f}{\partial x^2}$  at stationary points";
values = Table[ Apply[Evaluate[hesjanandfbisxhasz[[r]]] &, list[[r]], {1}],
  {r, 1, Length[listofrighthandside]}];
Do[ {Print[Style[r, 18, Red]],
Print["Function:"],
Print[ f[x, y] == ReleaseHold[ listoflefthandside[[r]] ] ],
Print["Stationary points:"], Print[ list[[r]] ],

Print["Hessian det and  $\frac{\partial^2 f}{\partial x^2}$ :"], Print[hesjanandfbisx[[r]] ],

Print["Values of Hessian det and  $\frac{\partial^2 f}{\partial x^2}$  at stationary points:"],
Print[ values[[r]] ] }, {r, 1, Length[listoflefthandside]}]]

```

In[]:=

ForTeachers[listofparameters]

1

Function:

$$f[x, y] = 6x^2 + x^3 + y^2 - xy^2$$

Stationary points:

$$\{\{1, -\sqrt{15}\}, \{1, \sqrt{15}\}, \{0, 0\}, \{-4, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 - 2x)(12 + 6x) - 4y^2, 12 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$\{-60, 18\}, \{-60, 18\}, \{24, 12\}, \{-120, -12\}\}$

2

Function:

$$f[x, y] = 3x^2 + x^3 + 4y^2 + 2xy^2$$

Stationary points:

$\{-2, 0\}, \{-2, 0\}, \{0, 0\}, \{-2, 0\}\}$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8 + 4x)(6 + 6x) - 16y^2, 6 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$\{0, -6\}, \{0, -6\}, \{48, 6\}, \{0, -6\}\}$

3

Function:

$$f[x, y] = 6x^2 + x^3 + y^2 + xy^2$$

Stationary points:

$\{-1, -3\}, \{-1, 3\}, \{0, 0\}, \{-4, 0\}\}$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 + 2x)(12 + 6x) - 4y^2, 12 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$\{-36, 6\}, \{-36, 6\}, \{24, 12\}, \{72, -12\}\}$

4

Function:

$$f[x, y] = 3x^2 + x^3 + 4y^2 - 2xy^2$$

Stationary points:

$\{2, -2\sqrt{3}\}, \{2, 2\sqrt{3}\}, \{0, 0\}, \{-2, 0\}\}$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8 - 4x)(6 + 6x) - 16y^2, 6 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$\{-192, 18\}, \{-192, 18\}, \{48, 6\}, \{-96, -6\}\}$

5

Function:

$$f[x, y] = 6x^2 + x^3 + y^2 + xy^2$$

Stationary points:

$\{-1, -3\}, \{-1, 3\}, \{0, 0\}, \{-4, 0\}\}$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2+2x)(12+6x)-4y^2, 12+6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-36, 6\}, \{-36, 6\}, \{24, 12\}, \{72, -12\}$$

6

Function:

$$f[x, y] = 6x^2 + x^3 + 4y^2 + 2xy^2$$

Stationary points:

$$\{-2, -\sqrt{6}\}, \{-2, \sqrt{6}\}, \{0, 0\}, \{-4, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8+4x)(12+6x)-16y^2, 12+6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-96, 0\}, \{-96, 0\}, \{96, 12\}, \{96, -12\}$$

7

Function:

$$f[x, y] = 6x^2 + x^3 + 4y^2 - 2xy^2$$

Stationary points:

$$\{2, -3\sqrt{2}\}, \{2, 3\sqrt{2}\}, \{0, 0\}, \{-4, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8-4x)(12+6x)-16y^2, 12+6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-288, 24\}, \{-288, 24\}, \{96, 12\}, \{-288, -12\}$$

8

Function:

$$f[x, y] = 9x^2 + x^3 + y^2 + xy^2$$

Stationary points:

$$\{-1, -\sqrt{15}\}, \{-1, \sqrt{15}\}, \{0, 0\}, \{-6, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2+2x)(18+6x)-4y^2, 18+6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-60, 12\}, \{-60, 12\}, \{36, 18\}, \{180, -18\}$$

9

Function:

$$f[x, y] = 9x^2 + x^3 + y^2 - xy^2$$

Stationary points:

$$\{\{1, -\sqrt{21}\}, \{1, \sqrt{21}\}, \{0, 0\}, \{-6, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 - 2x)(18 + 6x) - 4y^2, 18 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{-84, 24\}, \{-84, 24\}, \{36, 18\}, \{-252, -18\}\}$$

10

Function:

$$f[x, y] = 6x^2 + x^3 + 9y^2 + 3xy^2$$

Stationary points:

$$\{\{-3, -\sqrt{3}\}, \{-3, \sqrt{3}\}, \{0, 0\}, \{-4, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(12 + 6x)(18 + 6x) - 36y^2, 12 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{-108, -6\}, \{-108, -6\}, \{216, 12\}, \{72, -12\}\}$$

11

Function:

$$f[x, y] = 9x^2 + x^3 + 4y^2 + 2xy^2$$

Stationary points:

$$\{\{-2, -2\sqrt{3}\}, \{-2, 2\sqrt{3}\}, \{0, 0\}, \{-6, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8 + 4x)(18 + 6x) - 16y^2, 18 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{-192, 6\}, \{-192, 6\}, \{144, 18\}, \{288, -18\}\}$$

12

Function:

$$f[x, y] = 9x^2 + x^3 + 9y^2 + 3xy^2$$

Stationary points:

$$\{\{-3, -3\}, \{-3, 3\}, \{0, 0\}, \{-6, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(18 + 6x)^2 - 36y^2, 18 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-324, 0\}, \{-324, 0\}, \{324, 18\}, \{324, -18\}$$

13

Function:

$$f[x, y] = 12x^2 + x^3 + y^2 + xy^2$$

Stationary points:

$$\{-1, -\sqrt{21}\}, \{-1, \sqrt{21}\}, \{0, 0\}, \{-8, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 + 2x)(24 + 6x) - 4y^2, 24 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-84, 18\}, \{-84, 18\}, \{48, 24\}, \{336, -24\}$$

14

Function:

$$f[x, y] = 12x^2 + x^3 + y^2 - xy^2$$

Stationary points:

$$\{1, -3\sqrt{3}\}, \{1, 3\sqrt{3}\}, \{0, 0\}, \{-8, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 - 2x)(24 + 6x) - 4y^2, 24 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-108, 30\}, \{-108, 30\}, \{48, 24\}, \{-432, -24\}$$

15

Function:

$$f[x, y] = 12x^2 + x^3 + 9y^2 + 3xy^2$$

Stationary points:

$$\{-3, -\sqrt{15}\}, \{-3, \sqrt{15}\}, \{0, 0\}, \{-8, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(18 + 6x)(24 + 6x) - 36y^2, 24 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-540, 6\}, \{-540, 6\}, \{432, 24\}, \{720, -24\}$$

16

Function:

$$f[x, y] = 9x^2 + x^3 + 16y^2 + 4xy^2$$

Stationary points:

$$\{\{-4, -\sqrt{6}\}, \{-4, \sqrt{6}\}, \{0, 0\}, \{-6, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(18 + 6x)(32 + 8x) - 64y^2, 18 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{-384, -6\}, \{-384, -6\}, \{576, 18\}, \{288, -18\}\}$$

17

Function:

$$f[x, y] = 6x^2 + x^3 + 16y^2 + 4xy^2$$

Stationary points:

$$\{\{-4, 0\}, \{-4, 0\}, \{0, 0\}, \{-4, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(12 + 6x)(32 + 8x) - 64y^2, 12 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{0, -12\}, \{0, -12\}, \{384, 12\}, \{0, -12\}\}$$

18

Function:

$$f[x, y] = 12x^2 + x^3 + 9y^2 - 3xy^2$$

Stationary points:

$$\{\{3, -\sqrt{33}\}, \{3, \sqrt{33}\}, \{0, 0\}, \{-8, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(18 - 6x)(24 + 6x) - 36y^2, 24 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{-1188, 42\}, \{-1188, 42\}, \{432, 24\}, \{-1584, -24\}\}$$

19

Function:

$$f[x, y] = 9x^2 + x^3 + 16y^2 - 4xy^2$$

Stationary points:

$$\{\{4, -\sqrt{30}\}, \{4, \sqrt{30}\}, \{0, 0\}, \{-6, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(32 - 8x)(18 + 6x) - 64y^2, 18 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-1920, 42\}, \{-1920, 42\}, \{576, 18\}, \{-1440, -18\}$$

20

Function:

$$f[x, y] = -6x^2 + x^3 + 16y^2 - 4xy^2$$

Stationary points:

$$\{4, 0\}, \{4, 0\}, \{0, 0\}, \{4, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(32 - 8x)(-12 + 6x) - 64y^2, -12 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{0, 12\}, \{0, 12\}, \{-384, -12\}, \{0, 12\}$$

21

Function:

$$f[x, y] = 12x^2 + x^3 + 4y^2 + 2xy^2$$

Stationary points:

$$\{-2, -3\sqrt{2}\}, \{-2, 3\sqrt{2}\}, \{0, 0\}, \{-8, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8 + 4x)(24 + 6x) - 16y^2, 24 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-288, 12\}, \{-288, 12\}, \{192, 24\}, \{576, -24\}$$

22

Function:

$$f[x, y] = 12x^2 + x^3 + 4y^2 + 2xy^2$$

Stationary points:

$$\{-2, -3\sqrt{2}\}, \{-2, 3\sqrt{2}\}, \{0, 0\}, \{-8, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8 + 4x)(24 + 6x) - 16y^2, 24 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-288, 12\}, \{-288, 12\}, \{192, 24\}, \{576, -24\}$$

23

Function:

$$f[x, y] = 12x^2 + x^3 + 4y^2 - 2xy^2$$

Stationary points:

$$\{ \{2, -\sqrt{30}\}, \{2, \sqrt{30}\}, \{0, 0\}, \{-8, 0\} \}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{ (8 - 4x)(24 + 6x) - 16y^2, 24 + 6x \}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{ \{-480, 36\}, \{-480, 36\}, \{192, 24\}, \{-960, -24\} \}$$

24

Function:

$$f[x, y] = 12x^2 + x^3 + 16y^2 + 4xy^2$$

Stationary points:

$$\{ \{-4, -2\sqrt{3}\}, \{-4, 2\sqrt{3}\}, \{0, 0\}, \{-8, 0\} \}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{ (24 + 6x)(32 + 8x) - 64y^2, 24 + 6x \}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{ \{-768, 0\}, \{-768, 0\}, \{768, 24\}, \{768, -24\} \}$$

25

Function:

$$f[x, y] = 12x^2 + x^3 + 16y^2 - 4xy^2$$

Stationary points:

$$\{ \{4, -6\}, \{4, 6\}, \{0, 0\}, \{-8, 0\} \}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{ (32 - 8x)(24 + 6x) - 64y^2, 24 + 6x \}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{ \{-2304, 48\}, \{-2304, 48\}, \{768, 24\}, \{-2304, -24\} \}$$

26

Function:

$$f[x, y] = 15x^2 + x^3 + y^2 - xy^2$$

Stationary points:

$$\{ \{1, -\sqrt{33}\}, \{1, \sqrt{33}\}, \{0, 0\}, \{-10, 0\} \}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 - 2x)(30 + 6x) - 4y^2, 30 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-132, 36\}, \{-132, 36\}, \{60, 30\}, \{-660, -30\}$$

27

Function:

$$f[x, y] = 15x^2 + x^3 + y^2 + xy^2$$

Stationary points:

$$\{-1, -3\sqrt{3}\}, \{-1, 3\sqrt{3}\}, \{0, 0\}, \{-10, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(2 + 2x)(30 + 6x) - 4y^2, 30 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-108, 24\}, \{-108, 24\}, \{60, 30\}, \{540, -30\}$$

28

Function:

$$f[x, y] = 15x^2 + x^3 + 4y^2 + 2xy^2$$

Stationary points:

$$\{-2, -2\sqrt{6}\}, \{-2, 2\sqrt{6}\}, \{0, 0\}, \{-10, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(8 + 4x)(30 + 6x) - 16y^2, 30 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-384, 18\}, \{-384, 18\}, \{240, 30\}, \{960, -30\}$$

29

Function:

$$f[x, y] = 15x^2 + x^3 + 9y^2 + 3xy^2$$

Stationary points:

$$\{-3, -\sqrt{21}\}, \{-3, \sqrt{21}\}, \{0, 0\}, \{-10, 0\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(18 + 6x)(30 + 6x) - 36y^2, 30 + 6x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{-756, 12\}, \{-756, 12\}, \{540, 30\}, \{1260, -30\}$$

30

Function:

$$f[x, y] = 15 x^2 + x^3 + 16 y^2 + 4 x y^2$$

Stationary points:

$$\{\{-4, -3\sqrt{2}\}, \{-4, 3\sqrt{2}\}, \{0, 0\}, \{-10, 0\}\}$$

Hessian det and $\frac{\partial^2 f}{\partial x^2}$:

$$\{(30 + 6 x) (32 + 8 x) - 64 y^2, 30 + 6 x\}$$

Values of Hessian det and $\frac{\partial^2 f}{\partial x^2}$ at stationary points:

$$\{\{-1152, 6\}, \{-1152, 6\}, \{960, 30\}, \{1440, -30\}\}$$

Clear all definitions

In[]:=

Clear[f, listofparameters, ForStudents, ForTeachers]