Teaching Mathematics in English tracks

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Didactic Innovations in Academic Mathematics
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Fields of studies

- Corporate Finance and Accounting
- Global Finance and Accounting (with CFA certification)
  – both oriented on financial analysis, time series, etc.

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- Modern Business Management
  – methods applied in statistical analysis and management models

Students

- Poles
  - majority
  - most at basic level of Matura Exam
- Ukrainians
  - majority in minority
  - highly differentiated skills: from advanced to quite low
- Koreans, Chinese, Vietnamese, Indians
  - at most one-two students per year
- Nigerians
  - at most three-four per year

Challenges

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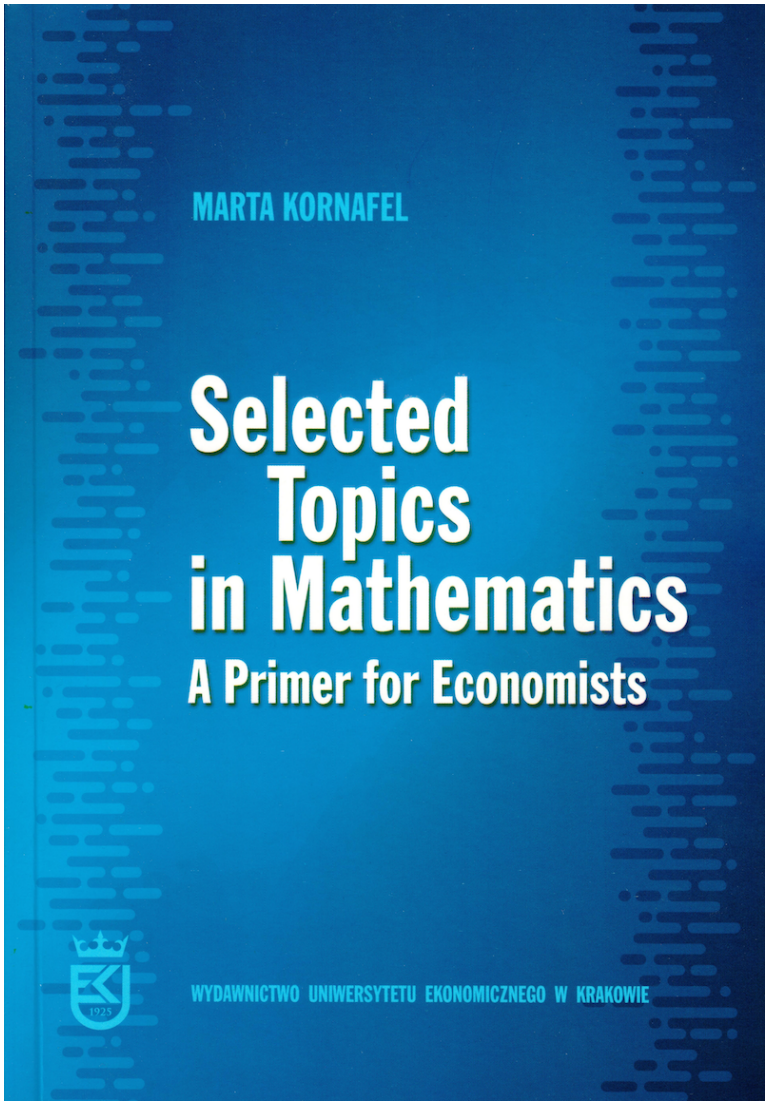
- highly differentiated background in school mathematics
- different approaches to responsibilites
- cultural implications

Typical lacks

- operations on algebraic expressions
- limited types of functions, usually: $y = ax + b$, $y = ax^2 + bx + c$, $y = 1/x$
- limited ability of reading graphs
- ungounded knowledge on probablity

Book (2019)



Contents

The tools – Formulas Questions quizzes at moodle

Question 2 Partially correct Mark 3.78 out of 6.00 Flag question Edit question

Find the inverse matrix of: $\mathbf{A} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$.

In the cells below write respective numbers or "none".

$\det \mathbf{A} =$ -1 .

so the inverse matrix exists true (*write here "true" or "false"*).

One possible correct answer is: -1, true

The cofactor matrix is:

$\mathbf{D_A} = \begin{bmatrix} -1 & 3 & 1 \\ -2 & 7 & -3 \\ -1 & 2 & -2 \end{bmatrix}$

One possible correct answer is: -1, 3, -1, -2, 7, -3, -1, 2, -1

Finally, the inverse matrix is:

$\mathbf{A}^{-1} = \begin{bmatrix} -1 & -2 & -1 \\ 3 & 7 & 2 \\ 1 & -3 & -2 \end{bmatrix}$

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Question 1 Partially correct Mark 4.17 out of 5.00 Flag question Edit question

Given the supply function $S(p) = \sqrt{2p + 250}$ in dependence on price p. Calculate the marginal supply for the price $p_0 = 3$ and interpret the result.

1) The marginal supply is: $MS(p) =$ sqrt(2 p+252)-sqrt(2 p+250)

✓

One possible correct answer is: 2/(2*sqrt(2*p+250))

2) The value of marginal supply for the price p_0 is:

$MS(p_0) =$ sqrt(258)-sqrt(256)

✓

One possible correct answer is: 0.06

3) Interpretation of the result:

if the price increases by 3 unit, then the supply increases by sqrt(258)-sqrt(256) percent.

✓

One possible correct answer is: price, increases, 1, unit, supply, increases, 0.06, unit

The tools – Formulas Questions quizzes at moodle

- Training quiz (1) - Logic
- Training quiz (2) - Properties of functions
- Training quiz (3) - Limit of sequence
- Training quiz (4) - Limit of function
- Training quiz (5) - Asymptotes
- Training quiz (6) - Continuity
- Training quiz (7) - Calculating derivatives
- Training quiz (8) - Marginality and elasticity
- Training quiz (9) - Approximation
- Training quiz (10) - Investigation of function

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The tools – Formulas Questions quizzes at moodle

Remarks on introducing the results in moodle

1. Number π = Pi. Number e is not recognised, it should be written as e = exp(1) (see below the remark about the function exp(x)).
2. $\sqrt{2}$ = sqrt(2)
3. write decimal fraction as 1.2 (use dot, not comma). If the result has a long decimal extension,, provide the it with accuracy up to 2 places, with usual rules or rounding, e.g. $1.2345 \approx 1.23$, $1.2365 \approx 1.24$
4. in the algebraic expressions use "*" or "space" for **multiplication**. System reads "ab" as the object "ab", while "a*b" or "a b" as the product $a \cdot b$. Examples: "8 x", "8*x", "8 sqrt(2)", "8*sqrt(2)".
5. System is not perfect. If you see "$+-2$" please treat it as "-2". If you see "$--2$" please treat it as "$+2$". Sometimes it happens that system shows "-0" as an answer. Ignore it, this is just zero.
6. If the answer is an algebraic expression, then any (correct) transformed form of it is accepted. E.g. if the answer is $(a+b)^2$, then any form: (a+b)^2, a^2+2*a*b+b^2, a*(a+2*b)+b*b is accepted and full mark for it is given.
7. Do not leave any cell for answers empty. If there is no value to put in a cell, write there "**none**".
8. Mathematical functions: sin(x), cos(x), tangent tan(x) (not $tg(x)$); **exponential** functions: e^x = exp(x), a^x = a^x, root: $\sqrt{x}$ = sqrt(x), absolute value: $|x|$ = abs(x), **logarithm**: $\ln(x)$ = ln(x)
9. If you need to enter a logarithm with another base than natural, use change-of-base formula:
   $\log_a(b) = \frac{\ln(b)}{\ln(a)}$
10. In the moodle answers in quizes the log(x) is displayed instead of ln(x). However, in the answer fields ln(x) should be used. The reason for it is that log(x) is identified with ln(x) in english systems, and our moodle "deeply inside" is written in English, but no correction of the name of function was made. Please be aware of it and apply the command appropriately, as said here.
11. Fractional expressions: (a+b)/(c+d)= $\frac{a+b}{c+d}$, a+b/c+d= $a + \frac{b}{c} + d$. If you need to write a simple fraction $\frac{a}{b}$, it is enough to write a/b (e.g. 2/3, 5/2, x/2, x^2/2).
12. Power a^b can be written as **a^b**. The only exception is power of Euler number - in this case the following convention should be used: e^3 = **exp(3)**.
13. To denote infinity, write "**inf**". Both, "inf" and "+inf" works for $+\infty$, while "-inf" works for $-\infty$.

The tools – guides at moodle: students for students

Functions of 2 variables

Function of 2 variables - local extrema and the least squares method

Function of 2 variables - partial derivatives, economic interpretations and approximation

Function of 2 variables - contrained extrema and the linear programming

5. Limits with number e

$$\lim_{n\to\infty}\left(1+\frac{3}{n+2}\right)^{n-2}=\lim_{n\to\infty}\left(\left(1+\frac{1}{\frac{n+2}{3}}\right)^{\frac{n-2}{3}}\right)^{3}=$$

$$=\lim_{n\to\infty}\left(\left(1+\frac{1}{\frac{n+2}{3}}\right)^{\frac{n+2}{3}}\right)^{3}\cdot\left(1+\frac{1}{\frac{n+2}{3}}\right)^{4}=e^{3}$$

$$\lim_{n\to\infty}\left(\frac{2n+1}{2n+2}\right)^{2n+2}=\lim_{n\to\infty}(\frac{2n+2}{2n+2}-\frac{1}{2n+2})^{2n+2}=\lim_{n\to\infty}(1+\frac{-1}{2n+2})^{2n+2}=e^{-1}=\frac{1}{e}$$

$$\lim_{n\to\infty}\left(\frac{2n+1}{2n+2}\right)^{n+1}=\lim_{n\to\infty}(\frac{2n+2}{2n+2}+\frac{-1}{2(n+1)})^{n+1}=\lim_{n\to\infty}[(1+\frac{-1}{2(n+1)})^{2(n+1)}]^{\frac{1}{2}}=$$

$$=(e^{-1})^{\frac{1}{2}}=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$$

The tools – guides at moodle: students for students

2. How to investigate extrema of function?

1. Extrema of function

Extrema of function – maximal and minimal value of the function.

Minima and maxima -

are the largest and smallest value of the function, either within a given range (the local or relative extrema, they determinate the monotinicty) or on the entire domain of a function (the global or absolute extrema).

2. Examples

- $f(x) = 2x^6 - \frac{12}{5}x^5 + 3x^4 - 4x^2 + 2$

$D = \mathbb{R}$

$f'(x) = 12x^5 - 12x^4 + 12x^3 - 12x^2$

Necessary condition: $f'(x) = 0 \Leftrightarrow 12x^5 - 12x^4 + 12x^3 - 12x^2 = 0$

$$12x^2(x^3 - x^2 + x - 1) = 0 \qquad | : 12$$

$$x = 0 \vee x^3 - x^2 + x - 1 = 0$$

$$x = 0 \vee (x - 1)(x^2 + 1) = 0$$

$$x = 0 \vee x - 1 = 0 \vee x^2 + 1 = 0 \rightarrow \text{always positive}$$

$$x = 1$$

$x_1 = 0$ and $x_2 = 1 \longrightarrow$ zero points of derivative

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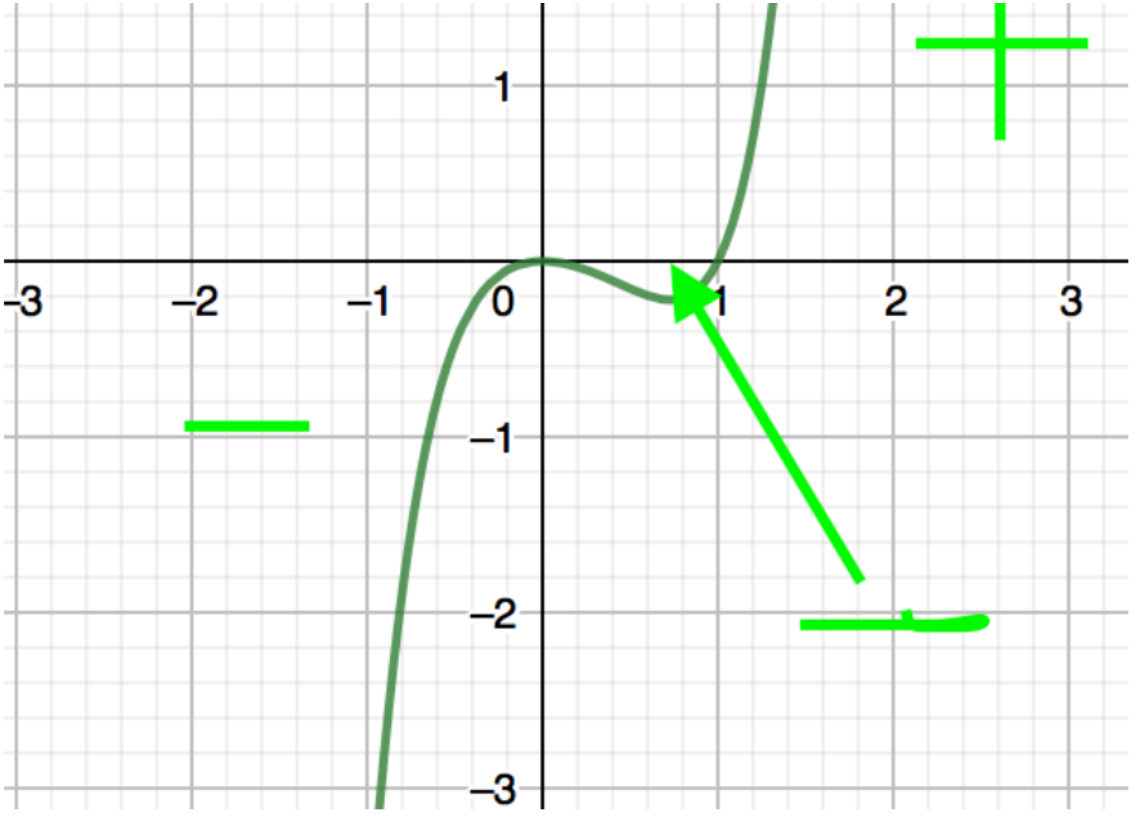

in the neighborhood of 0 $f'(x)$ does not change a sing $\Rightarrow$ no extremum in that point

x	(-∞,1)	{1}	(1,∞)
f'(x)	-	0	+
f(x)	↓	min	↑

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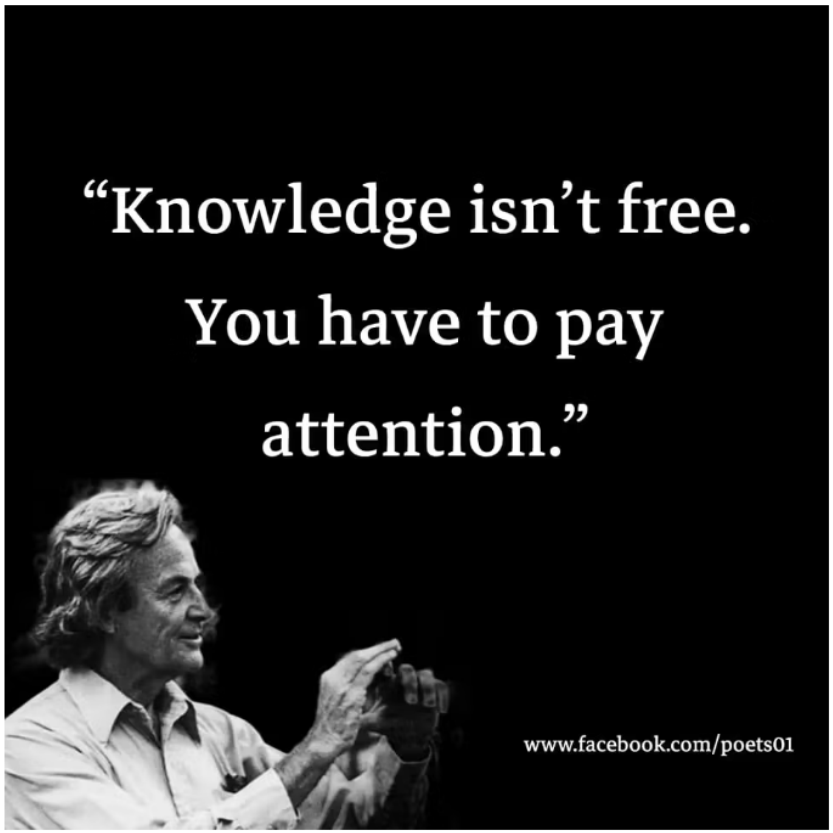

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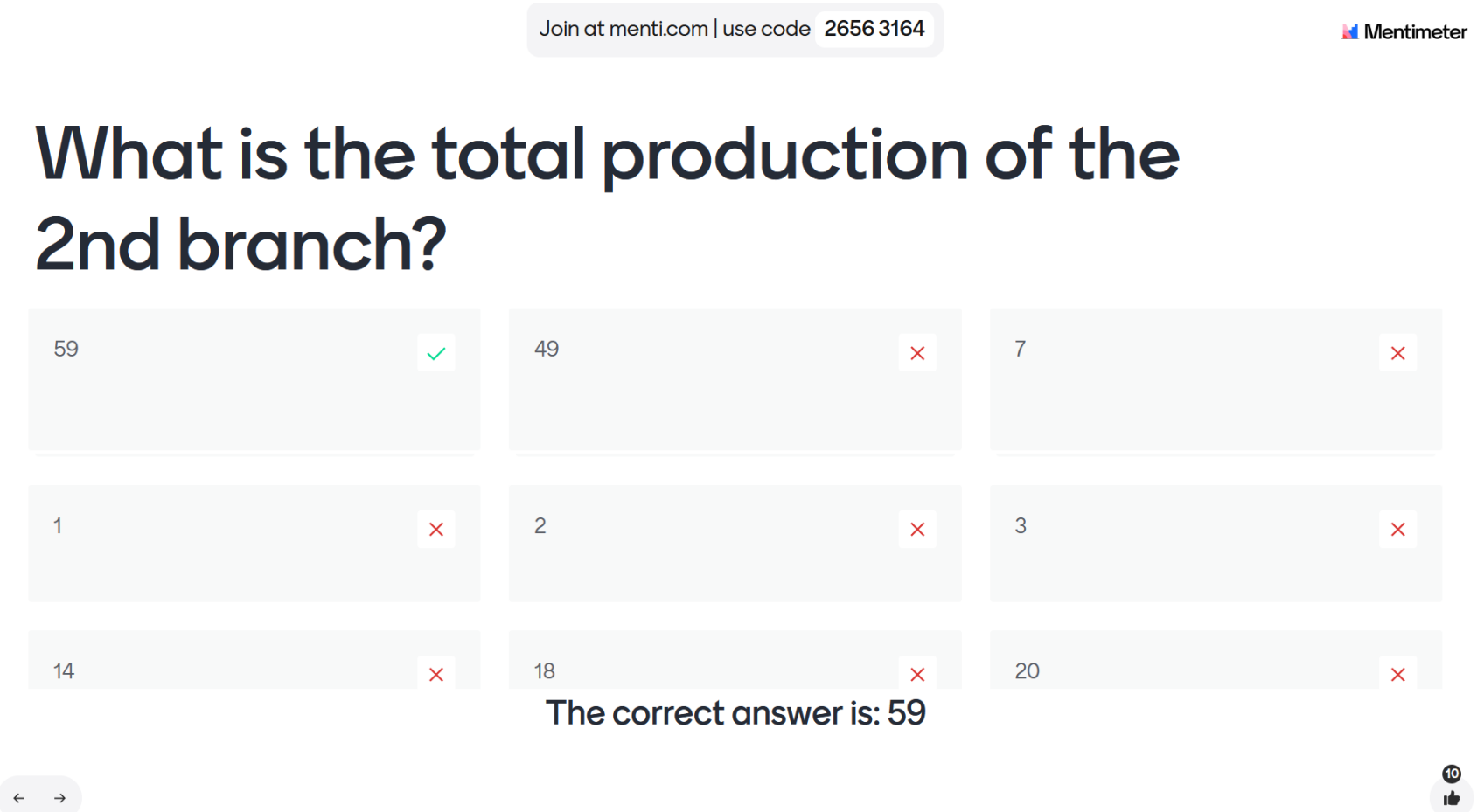

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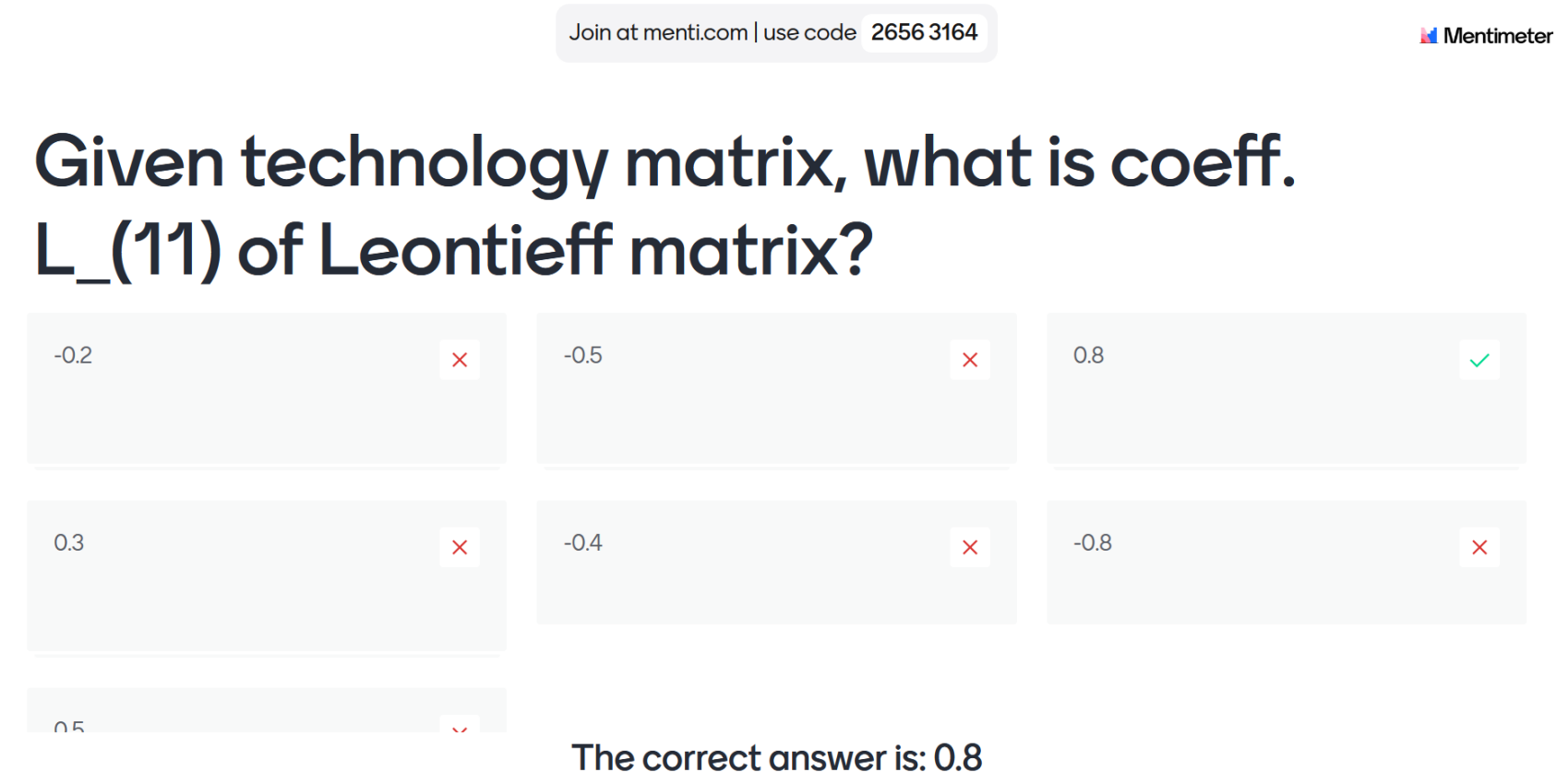

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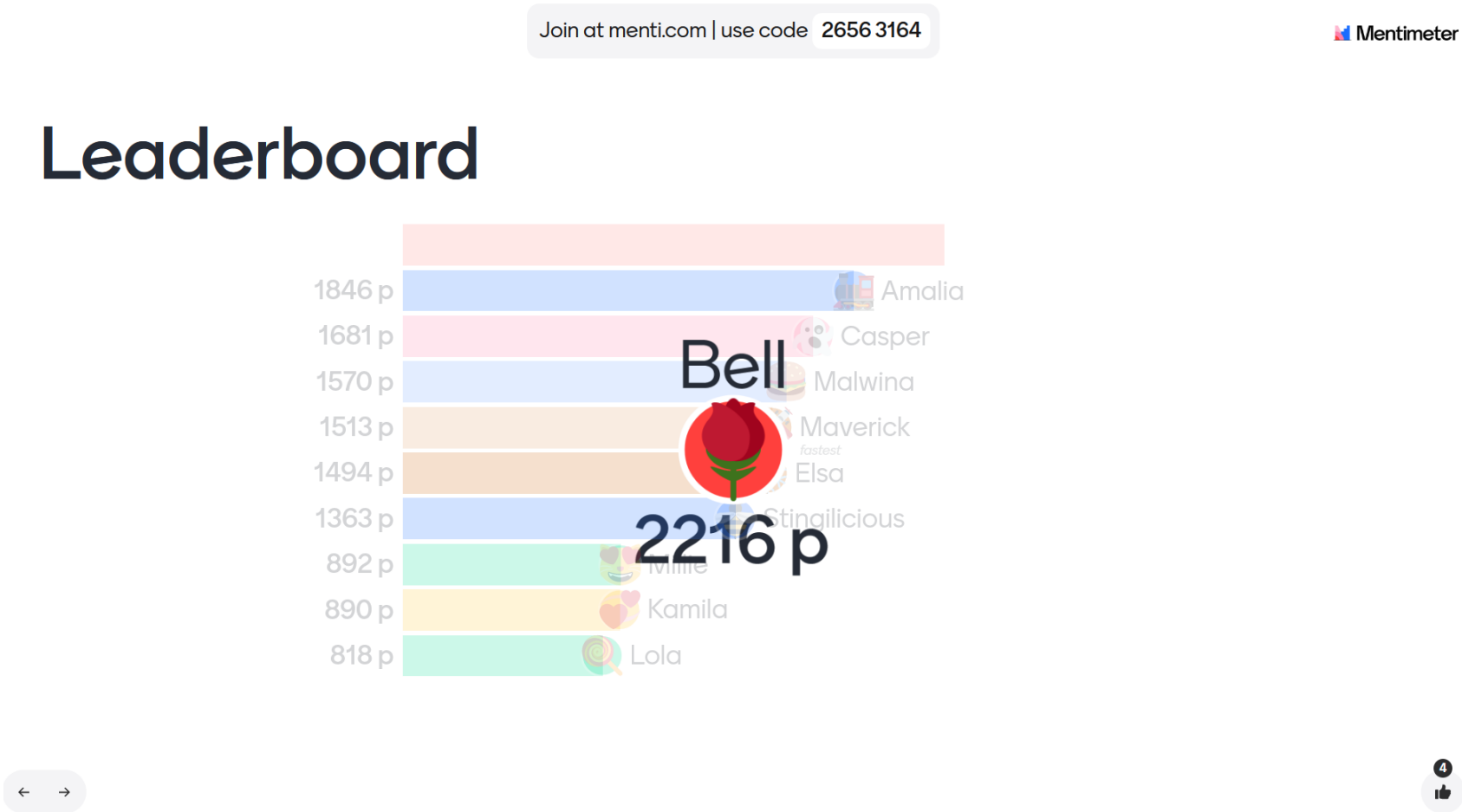

THANK YOU FOR YOUR ATTENTION